ON THE UNIQUENESS OF QUASIHYPERBOLIC MAGNETOHYDRODYNAMIC FLOWS

(O EDINSTVENNOSTI KVAZIGIPERBOLITCHESKIKH TECHENII MAGNITNOI GIDRODINAMIKI)

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Consider a thin body immersed in a uniform stream with velocity V of an ideal gas with infinite electric conductivity in presence of a magnetic field H parallel to V (Fig. 1). Such a flow is characterized by two dimensionless parameters: the Mach number M and the ratio N of Alfven's speed $H/\sqrt{4\pi\rho}$ to the speed of sound $\kappa p/\rho$.

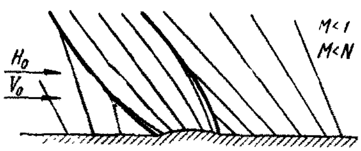

FIG. 1.

When $N/\sqrt{1 + N^2} < M < \min(1, N)$ (the quasihyperbolic region) the flow is of the hyperbolic type [1]. Kogan [1] gave the solution of the magnetohydrodynamic equations for the flow around the body subject to the conditions of zero flow-through at the body, of vanishing disturbances at infinity and of entropy increase in shock waves. In that solution shock waves propagate upstream as shown in Fig. 1.

We note that the now-established fact of upstream propagation of shock waves in subsonic hyperbolic flows [1] went unnoticed by a number of authors. Thus, in his study of magnetohydrodynamic shocks which is similar to that of [2], Cabannes [3], having found waves with angles exceeding 90°, remarks that, in his opinion, the waves cannot spread upstream and ascribes them to a flow angle greater than 180°, in which case the wave would spread downstream.

In [4] Sears reported on some results of Ressler's studies of gas flows past thin profiles with infinite electrical conductivity in the presence of a magnetic field parallel to the stream velocity. One and the same solution was presented for the subsonic and supersonic hyperbolic flows, i.e. it was assumed that in both cases the shock waves propagate downstream as in ordinary gasdynamics, which led to incorrect expressions for velocities and forces in the subsonic region.*

We will show that the solution of Fig. 1 is unique. In this flow field there are two families of characteristics corresponding to the inclinations

$$\tan\theta = \pm\sqrt{\frac{M^2 - N^2(1-M^2)}{(1-M^2)(N^2-M^2)}} \tag{1}$$

The characteristics with positive values of tan θ will be considered as belonging to the first family. The relationships valid along the characteristics have the form [1]

$$\mp(M^2 - N^2)\,|\tan\theta\,|\,d\vartheta - N^2 d\ln H - \frac{d\ln p}{\varkappa} = 0 \tag{2}$$

where ϑ is the angle of local velocity and p the pressure. When the disturbances are small, we can neglect changes in entropy (which are of the order of ϑ^3) and obtain

$$\mp(M^2 - N^2)\,|\tan\theta\,|\,d\vartheta + [M^2 - N^2(1-N^2)]\,d\ln V = 0 \tag{3}$$

Since in the last analysis M and N are functions of V, Equations (3) determine in the hodograph plane two families of curves which we shall call epicycloids in analogy with ordinary gasdynamics. As in the ordinary gasdynamical case, the shock polar and the epicycloids coincide in the linear and in the second-order approximations.

Consider an arbitrary point in the flow field or on the boundary of the body. Two characteristics pass through such a point. Altogether there are four possibilities:

a) Both characteristics come from infinity.

b) The characteristic of the first family comes from infinity downstream of the body, and the characteristic of the second family ends at the shock wave.

c) The characteristic of the first family ends at the shock wave and the characteristic of the second family goes to infinity upstream of the body.

d) Both characteristics run into the shock wave.

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* It is true, however, that a later version of the same report [5] contains a sentence calling attention to upstream propagation of flow disturbances. Yet the solutions themselves were not changed.

The question of uniqueness consists essentially of the clarification of which of the four cases occurs in reality.

In [1,2] it was shown that if the entropy increases in the quasi-hyperbolic flow field under study, the deflection of the stream corresponding to a positive angle ϑ occurs in shock waves with angles $\sigma > \pi/2$ (defined as a wave of second type). For shock waves with angles $\sigma < \pi/2$ the stream turns with a negative deflection angle ϑ (defined as a wave of first type).

A shock wave is completely determined when, alongside the scalar quantities which fix the strength of the wave, we are given the direction of propagation or of weakening of the shock. This direction can be ascertained from the characteristics running into the shock wave. For the present case, the nature of the approach of the characteristics to the shock wave is shown in Fig. 2, where the arrows indicate the direction of the shock.

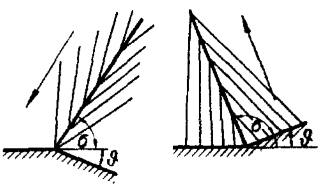

FIG. 2.

Let us suppose that in the flow under scrutiny there is a shock wave of the first type ($\sigma < \pi/2$). Then the characteristics of the first family running into the shock can come only from infinity; as waves of the second type these characteristics go through and cannot run into the shock wave of the first type again, as can be seen from the geometry.

Consequently, in the hodograph plane the points which correspond to velocities before and after the shock must both lie on the same epicycloid of the first family. On the other hand, velocities before and after the shock must lie on the shock polar, which to the adopted accuracy coincides with the epicycloid of the second family. Clearly, two points cannot lie simultaneously on two curves which intersect only once in the region under study. Therefore, outgoing shock waves of the first type cannot exist in the present flow field. Such shock waves can come toward the body only when caused by an extraneous generator.

Therefore, the third and fourth of the listed flow models cannot be realized. The first case appears trivial and can obviously be realized only in an undisturbed stream.

Consequently, the solution of [1] which is according to the flow model No. 2 is unique.

The disturbances which propagate upstream then correspond to a flow which appears undisturbed to first and second order downstream of the body. To a third order of approximation, the flow downstream of the body is disturbed because of the entropy rise across the shock wave.

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BIBLIOGRAPHY

1. Kogan, M.N., Magnetodinamika ploskikh i osesimmetrichnykh techenii gaza s bezkonechnoi elektricheskoi providimostiu (Magnetodynamics of two-dimensional and axisymmetric gas flows with infinite electrical conductivity). *PMM* Vol. 23, No. 1, 1959.

2. Kogan, M.N., Udarnye volny v magnitnoi gidrodinamike (Shock waves in magnetohydrodynamics). *PMM* Vol. 23, No. 3, 1959.

3. Cabannes, H., Dynamique des gaz ionisés: détermination des chocs stationnaires attachés à la pointe d'un dièdre. *Recherche aeronautique* No. 71, 1959.

4. Sears, W.R., Magnetohydrodynamics in aeronautics. *ARS, Paper 709-59; report at the meetings of the American Rocket Society,* Nov. 1958.

5. Sears, W.R., Magnetohydrodynamic effects in aerodynamic flows. *ARS Journal* Vol. 29, No. 6, 1959.

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